

Facilitating Controller Evolution in Morpho-functional-Machines – A Bipedal Case Study

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Abstract

Simulations of bipedal walking showed that the evolution of stable gaits can be greatly facilitated by harnessing design solutions inspired by those of biological organisms.

Three such areas are addressed in this paper: a) the use of passive dynamics such as free-swinging lower legs, knee caps and springy ankles, b) proportional derivative controllers (or their variants) as actuators, and c) a modular, specifically coupled neural controller architecture.

It is shown that an appropriate implementation of these components greatly improves the speed of evolution – stable straight line walking is typically achieved in substantially less than 100 generations. Moreover, compared to earlier experiments, the quality of the solutions arrived at is more satisfying with respect to their resemblance to human body dynamics during walking.

1 Introduction

Morpho-functional machines, whether embodied or simulated, typically achieve mobility through wheels or articulated limbs. While controllers for the former are relatively trivial, locomotion based on legs is a much harder task to accomplish. Here, recurrent neural networks have become a popular control architecture, usually paired with artificial evolution [1] to optimise the parameter settings [2].

Several researchers have used this combination successfully to control multi-legged morpho-functional machines in simulation [3, 4, 5] or embodied [6, 7, 8, 9]. More recently, the applicability of this approach was demonstrated to be extendable to the control of bipedal walking [10, 11].

Bipedal walking is a particularly difficult behaviour to control due to the inherent instability of two-legged locomotion. While an earlier evolutionary approach did produce stable walking cycles in simulation [11], only a small proportion (approximately 10%) of evolutionary runs resulted in successful controllers. Moreover, the evolved walking styles often did not closely resemble those found in natural bipeds, such as humans or birds.

It is argued here that these shortcomings can be overcome by implementing design solutions taken from biology. In particular these are:

- a) Passive body dynamics
- b) More responsive virtual muscle models
- c) Modular, coupled network architectures

It will be shown that the appropriate use of the above components leads to faster evolution and qualitatively more satisfying solutions.

This paper is structured as follows: a short background of bipedal walking (Section 2) will be followed by a description of the implementation used in this research (Section 3). Subsequently, the key components will be addressed in more detail in Section 4.1 (passive dynamics), 4.2 (muscle models) and 4.3 (network architecture). Sections 5 and 6 discuss the results of applying this improved architecture, with Section 7 concluding this work.

2 Bipedal Morpho-functional Machines - Background

Bipedal morpho-functional machines and robots have been subject to intensive research for several decades due to their potentially wide range of applications (e.g. as service robots) and because of their appeal as anthropomorphic machines. Two-legged robots with varying complexity have been produced and controlled by a number of researchers [e.g. 12, 13, 14, 15, 16, 17, 18]. Control is typically based on state-machines to produce the rhythmic leg movements, and additional controllers to maintain lateral and sagittal stability. Despite advances in computing power and mechanical implementation, even the best robots only achieve walking speeds that are considerably lower than those of a comparable human. Moreover, walking stability has yet to reach that of humans (see [19] for an overview of current bipedal walking machines).

The advent of readily available computing power has resulted in an increased trend to implement bipedal

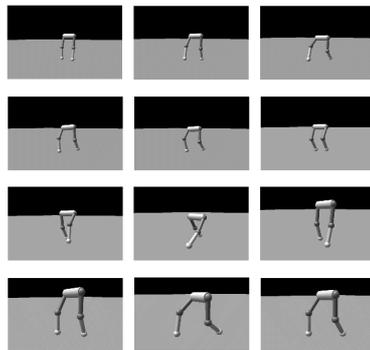


Fig.2. Motion sequence of biped controlled by an evolved RNN [10]



Fig. 1. Honda P3

MfMs in simulation. The attraction of this approach lies in the easy construction of a physically simulated machine, as compared to building one. Thus, different bodies and controller architectures can be tried and tested with relative ease. Pratt [20] has used this approach to construct a simulated bipedal walker modelled on MIT's M2 robot. Walking cycles are produced by state machines and stabilised by supplementary algorithms that control lateral foot placement and ankle torque. By manually tuning the corresponding parameters, this simulated robot was able to achieve average speeds of 0.8 m/s.

Reil [10] used a different approach to control a bipedal walker in simulation. The required cyclic actuator activities were not determined by a state machine, but produced by a biologically inspired central pattern generator (CPG), implemented as a recurrent neural net. Artificial evolution was used to optimise the controller's parameters. It was found that the genetic algorithm was capable of fine-tuning [21] the muscle activities in such a way that the walking cycles were stable without additional control algorithms for balance [11]. However, only moderate walking speeds of approximately 0.5 m/s could be achieved.

3 Implementation

The implementation used in this research is built upon the approach taken in [11], but incorporates several advances in the controller, actuator and body design.

3.1 Body

The biped is simulated using Mathengine's Dynamics Toolkit 0.0.5 [22], which includes the necessary functionality for rigid-body-dynamics and collision detection, and allows for significantly faster-than-real-time evaluation of controllers (Figure 3). Currently, the biped has 8 degrees of freedoms (DOF): hip-roll, hip-pitch, knee-pitch, ankle-pitch. Of these, only the two hip joints are actively actuated (see Section 4.1). The feet are implemented as two spheres each, representing the heel and toe sections.

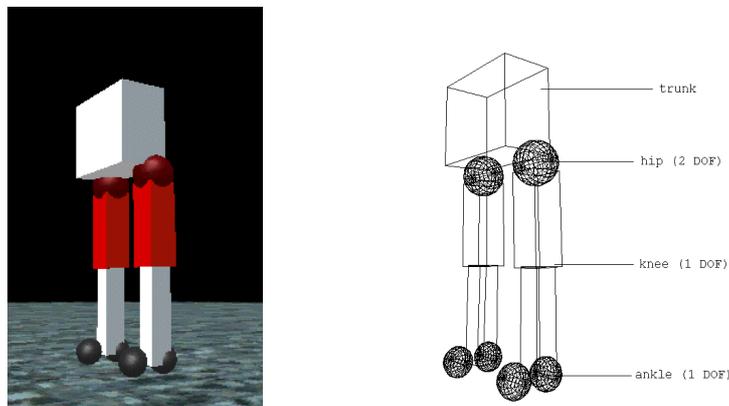


Fig. 3. Implementation of biped used in this research, fully rendered (left) and wire-frame (right).

Body	Implementation	Dimensions
HEEL	SPHERE	0.08m radius
TOE	SPHERE	0.08m radius
CALF	CUBOID	0.125m x 0.540m x 0.125m
THIGH	CUBOID	0.175m x 0.500m x 0.175m
HIP	SPHERE	0.125m radius
PELVIS	CUBOID	0.500m x 0.500m x 0.600m

Table 1. Physical Parameters of the Biped (density: 0.5kg/l for pelvis) and 1kg/l for other bodies)

3.2 Genetic Algorithm

The GA used to optimise the controller architecture is modified from [10].

The parameters to be evolved are synapse weights and node properties and are encoded as real numbers in a linear chromosome. The mutation rate is chosen so as to cause an average of one substitution per individual. No crossover operator is applied [11].

Rank based selection is used on a population of 100 individuals, with a fittest fraction of 0.5 (which is equivalent to culling the bottom half of the population and replacing it with a copy of the top half [23]).

Individuals are selected on the basis of the fitness function depicted in Figure 4. The overall fitness criterion is the distance of the hindmost foot to the origin of the walk (d). The hindmost foot is chosen as a reference point to penalise individuals that take only one great stride.

The fitness function decision tree of Figure 4 serves several functions. Most importantly, it enforces the early abortion of unpromising runs ($t < t_e$) to speed up evolution. An evaluation is terminated early if the controller has reached a point attractor ($\Delta a = 0$), and hence the body has ceased to move. In addition, a run is aborted if the position of the centre of mass (CoM) falls below a pre-defined height ($h < \beta$). This penalises falling down as well as grotesque movements [10]. Finally, individuals that fall over without previously moving ($d < \alpha$) are assigned a fitness of 0

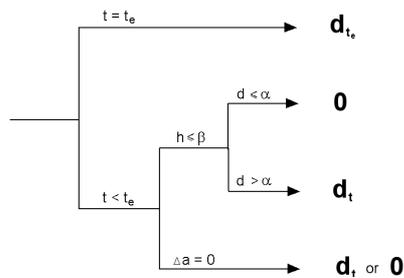


Fig. 4. Fitness function decision tree.

t : time, t_e maximum time, h : CoM height, β : CoM height threshold, α : distance threshold, Δa : RNN activation change, d : distance hindmost ankle/origin

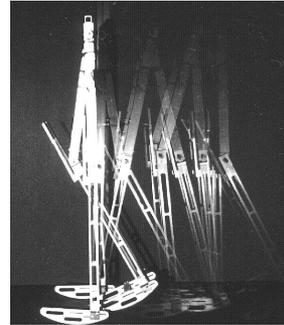


Fig. 5. Passive walker, based on McGeer's original design, Ruina Lab.

A weak assisting controller is used at the beginning of each evolutionary run in order to stabilise the trunk section laterally and sagittally. The controller serves to initially guide evolution towards the production of rhythmic step cycles, and is subsequently removed (see Section 5).

4 Exploiting Biological Design Solutions

Our implementation of the biped makes use of three distinctive classes of design solutions, with the purpose of facilitating the evolution of walking. These are detailed in the following subsections.

4.1 Passive Dynamics

The term passive dynamics describe the dynamic behaviour of a physical system (such as a biped) that is solely due to its mechanic properties.

Several workers have demonstrated that anthropomorphic walking machines can exhibit stable, human-like walking on a slope with no actuation and no control [24, 25, 26]. This has led researchers to postulate that bipedal walking “might be largely understood as a passive mechanical process” [27]. The reliance of slopes, and therefore gravity, as a power source for passive walkers indicates that this cannot be the full story – after all, biological bipeds are powered by muscles. However, the findings do suggest that natural selection may have tuned the morphology of biological bodies so as to significantly facilitate their control by the nervous system. Using a simulation of their M2 robot, Pratt & Pratt [28] have demonstrated that the exploitation of natural dynamics can indeed be coupled with controlled actuation of virtual muscles.

The biped presented here features mechanical components which serve to implement important passive dynamics as identified by earlier research projects [24, 25, 27, 28]. In particular, these are passively swinging lower legs, knee caps and springy ankles.

4.1.1 Passive Swing Legs

The leg movements characteristic of anthropomorphic walking are caused by a combination of actuation and passive swinging. In particular, in normally paced walking actuation seems to be largely limited to the hip joints, with the lower legs following passively. Given this, the control task for straight line walking can be simplified to two degrees of freedom (the hip joints) in an accordingly designed bipedal morpho-functional machine.

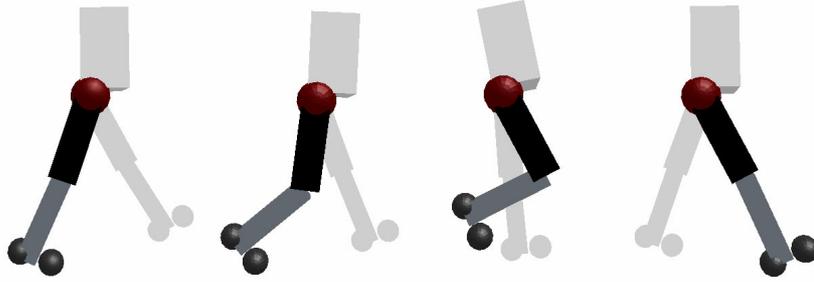


Fig. 6. Passive swing leg, one walking cycle.

The dynamic of a passive swing leg in a single walking cycle are depicted in Figure 6. Note that the passive lower leg is accelerated only by gravity and the pulling movement of the upper leg. In addition to simplifying the control (because the knee joint is not actively actuated), this implementation markedly increases the realism of the walking pattern produced.

4.1.2 Knee Cap

A passively swinging lower leg requires a mechanism to provide a rapid stop to the movement at full extension. An appropriately controlled and timed actuator could in principle achieve this functionality. In biological bipeds however, nature has opted for a cheaper, mechanical implementation in the form of a knee cap.

While a physical knee cap can be implemented relatively simply in an embodied robot, in simulation it is often cheaper to use a soft angular limit to the knee joint, which is the solution employed in the present study.

4.1.3 Compliant Ankle

The two feet represent the only physical interface between a walking biped and its environment, and therefore play an important role in its locomotion. In particular, by acting analogously to a damped spring, the ankles serve to allow a smooth roll of the foot over the floor while simultaneously storing energy. At toe lift-off, this energy is released and used to initiate the next stride. Again, no active neural control is required for this task. Our current biped includes this functionality in the form of a damped torsional spring in each of the ankle joints.

4.2 Actuators

While higher level control centres, such as spinal central pattern generators or the

neocortex, are often viewed as dominating the control of human and animal locomotion, a great deal of co-ordination is governed by simple lower level mechanisms [29, 30, 31]. Of particular importance in this respect are spinal reflexes, and of these the *stretch reflex* [32] is the most prominent example.

Skeletal muscles are extensively supplied with a variety of receptors. One type, muscle spindles, respond to stretch of muscle fibres and relay this information via Ia afferent neurones to the spinal alpha motor neurones. These in turn innervate the muscle concerned, as well as its antagonist. In the case of the former, neurone activity enhances muscle contraction; in the case of the latter, it inhibits it.

This set-up acts as a negative feedback controller. Its function is to counteract passive stretching of a muscle by actively increasing its degree of contraction.

In engineering terms, the stretch reflex is roughly equivalent to a proportional integral derivative (PID) controller. Applied to actuator control of MfMs, it can be expressed as follows:

$$T = k_P(\theta_d - \theta) + k_I \int (\theta_d - \theta) dt + k_D \dot{\theta}$$

T : torque force; coefficients: k_P : proportional, k_I : derivative, k_D : integral θ_d : desired angle, θ : actual angle

An actuator implementation of this type requires as input the desired angle (equivalent to the desired length of a biological muscle). The necessary force to reach the 'sweet spot' is calculated by the actuator itself. That is, motor commands from higher control centres (such as a CPG) to actuators do not represent forces, but desired kinematic parameters like relative joint angles.

The use of PID controllers brings about several advantages. They provide control at a low level, as they automatically reduce any discrepancy between the actual and the desired angle. As a consequence, the tendency of limbs, for example, to buckle under the body mass is counteracted directly by the actuator, a task which would otherwise have to be accomplished by the neural controller. In addition, by tuning the allowed desired angle (θ_d) range, upper and lower caps to limb movement can be easily achieved. This is not possible if the neural controller has to explicitly specify forces rather than desired angles.

Finally, a joint-actuator set-up that acts as one entity rather than implementing single muscles (agonists, synergists, antagonists) greatly reduces the degrees of freedom of the system (and of the neural controller). In vertebrates, the set of biological muscles around a joint act in a comparably integrated manner (Lloyd's myotatic unit [34]).

However, in practice, the use of PID controllers can be problematic. Firstly, changes in the control environment (e.g. load on the body) require dynamic parameter modulation of the PID. Because this modulation must itself be controlled, more DOFs are added to the system. Secondly, PID controllers can bring about numerical instabilities. For these reasons, we opted for an alternative actuator type called *force limited velocity constraints* (FLVCs), which provide a similar functionality to PIDs, but without the aforementioned shortcomings.

FLVCs allow the user to specify a relative velocity profile at which to approach the desired angle between two bodies. The simulation's constraint solver then calculates the forces required to satisfy this request. An external modulation of controller parameters is therefore unnecessary, even if the control environment changes.

4.3 Controller Architecture

The rhythmic activities required for vertebrate locomotion are known to be produced by dedicated neural structures in the spine, called central pattern generators (CPGs) [35]. While these circuits have been located in various animals, they remain elusive in humans (but see Bussel *et al.* [36] for indirect evidence).

Despite this lack of experimental evidence, recent theoretical studies have shed light on the CPG architecture necessary to produce the gait patterns displayed by quadrupeds and bipeds [37]. In essence, the proposed structure consists of two bi-directionally coupled chains of identical oscillators (Figure 7). The number of the latter is twice that of the appendages to be actuated (i.e. 8 for quadrupeds, 4 for bipeds).

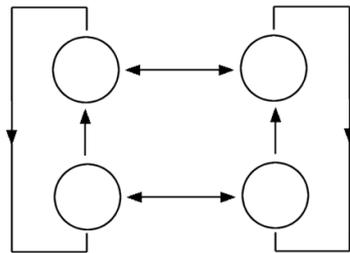


Fig. 7. Schematic illustration of CPG architecture proposed by Golubitsky [37]. The CPG for a biped is shown. Circles indicate oscillators. Arrows denote coupling directions.

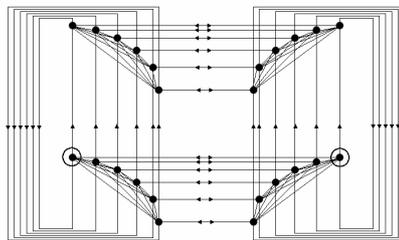


Fig. 8. Neural CPG architecture used in this research. Circled nodes are motor neurons. See text for details.

In the present work, this general set-up is translated into a neural implementation, as depicted in Figure 8.

Each sub-oscillator is represented by a fully connected recurrent neural net with six nodes¹. Each of these is connected to the laterally adjacent oscillator via symmetrical weights (bi-directional coupling), with each neurone synapsing with its equivalent in the opposite network. Connections to the sagittal neighbours are asymmetric, i.e. each neurone is connected to its equivalent in the next oscillator, but no connections exist the other way.

The neurone model is that used in [7, 10], and includes a bias and time parameter. Their allowed ranges, as well as those of the synapse weights, are detailed in [10]. Two nodes are designated motor neurones - their output is used for left and right sagittal hip actuation (Figure 8).

The use of identical oscillators as well as only two sets of coupling weights (lateral and sagittal) limits the search space to a total of 60 parameters (36 weights + 12 neurone parameters + 6 lateral weights + 6 sagittal weights).

5 Results

Evolutionary runs resulted in realistic straight line walking in a small number of generations (Figure 9).

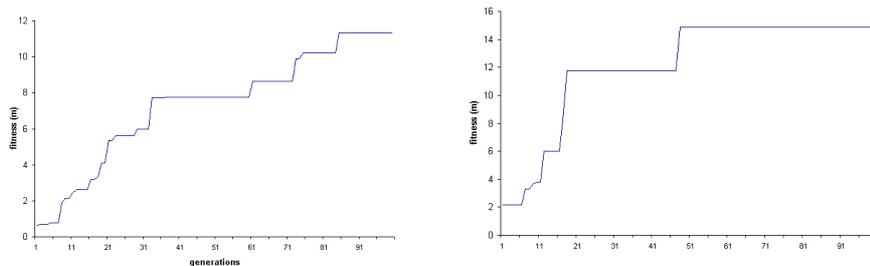


Fig. 9. Two representative evolutionary runs leading to successful straight-line walking. Assisting controller active.

Because initial runs were performed with a weak stabilising assisting controller (see Section 3.2), populations of stable walkers were picked at random from a series of evolutionary runs and re-evolved with the assisting controller switched off. It was found that these individuals reliably evolved to attain the previously achieved fitness (Figure 10). Control experiments with *de novo* evolution and de-activated ancillary controllers did not result in successful runs (Figure 11).

¹ Six nodes were chosen as these were found to produce the highest degree of spontaneous cyclic activity.

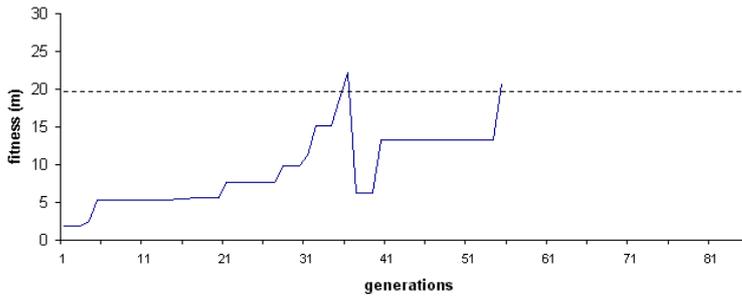
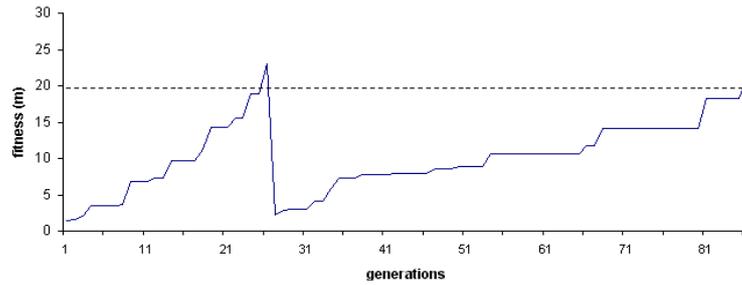


Fig. 10. Incremental evolution. Runs are evolved with ancillary controllers until the walker reaches 20 m. The ancillary controller is then switched off and the population is re-evolved. Two runs (top, bottom) are shown.

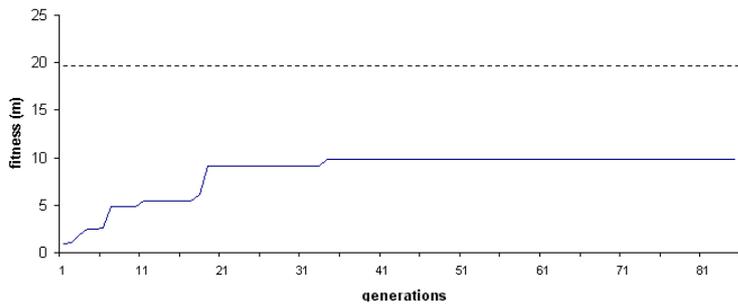


Fig. 11. Evolutionary run without active ancillary controller. The milestone fitness of 20 m is not reached.

Evolved walkers predominantly displayed a standard walking gait which made full use of the passive dynamics of the body, such as free swinging legs (Figure 12). However, a small number of evolutionary runs resulted in small-step gaits, with almost no lower leg swing. The average walking speed of walkers was 0.8 m/s, with maximum speeds of approximately 1.2 m realised by several individuals.

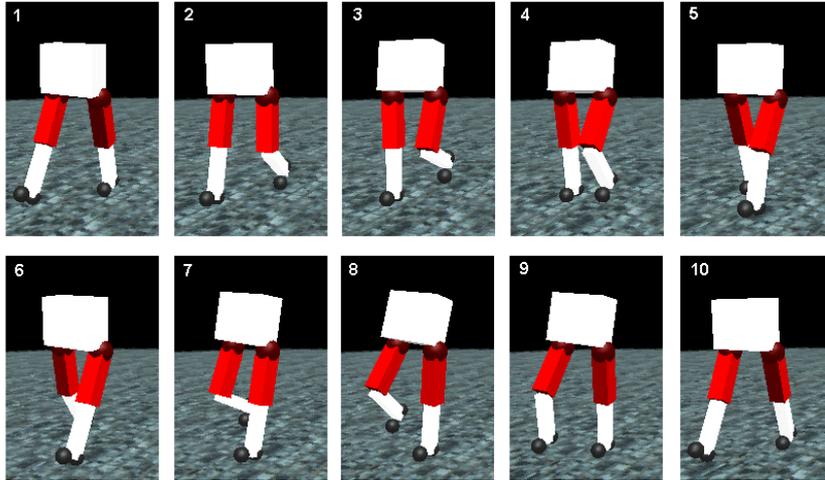


Fig. 12. Frame shots of an evolved walker. One gait cycle is shown.

Analysis of neural activity and resulting body dynamics showed that relatively simple periodic controller outputs sufficed to produce stable walking (Figure 13). The periods of neural output and body dynamics (e.g. hip angles) were locked (Figure 13 and 14). Existing sub-oscillations in the controller output (as exemplified in Figure 13) were not reflected in the overall body dynamics (this is because the lag between actual and desired angles acts like a low pass filter, smoothing out small oscillations in the desired angle trace).

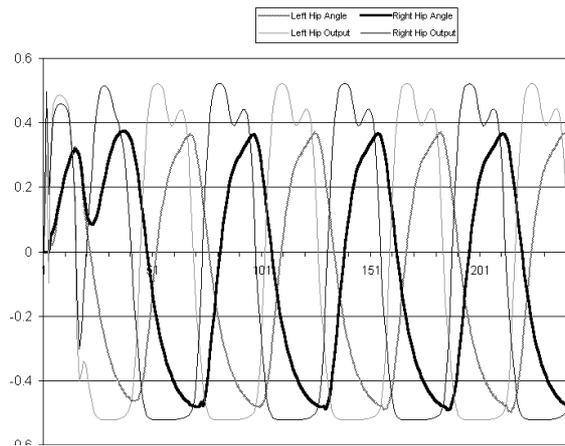


Fig. 13. Time series of neural output (desired left and right hip angles in radians) and resulting body dynamics (actual left and right hip angles in radians). Note that the small sub-oscillation in the neural output is not reflected in the body dynamics. (Data taken from individual depicted in Figure 12.)

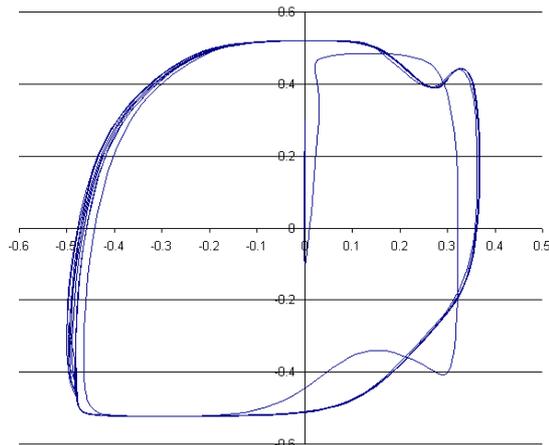


Fig. 14. Left hip angle plotted vs. left hip neural output. After an initial settling phase, both periods become locked to each other.

Periodic gaits were always preceded by a short settling phase, in which the neural outputs and body dynamics differed from the subsequently established pattern (Figure 15). Visual analysis showed that this stage was used to initialise the body position for the walking gait.

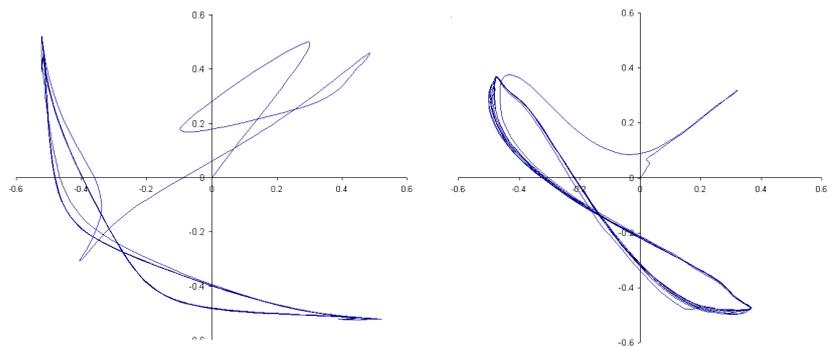


Fig. 15. Attractor plots of left/right hip desired angles (left) and left/right hip actual angles (right). Note that both trajectories exhibit an initial settling phase at the onset of walking. Angles are shown in radians.

An initial settling phase can be observed even more prominently in the resulting passive body dynamics. Figure 17 shows the left knee / left hip angle trajectory of the walker displayed in Figure 13. Note that it takes several steps until the knee reaches its final full swing cycle.

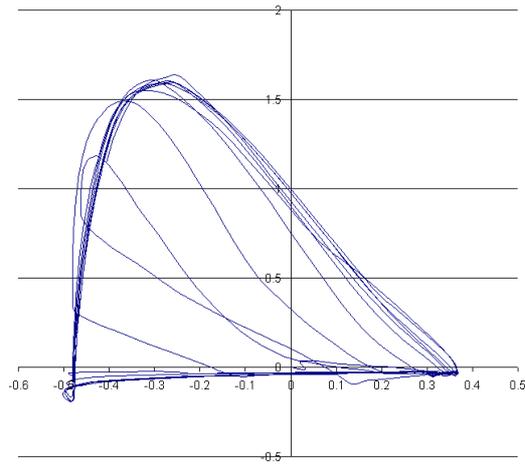


Fig. 16. Left hip angle (actuated) vs. left knee angle (passive). The stable attractor cycle is reached after several steps. Angles are shown in radians.

6 Discussion

The fitness graphs of Figure 10 show that the length of evolutionary runs required to produce straight line walking is very short. Moreover, the large majority of runs result in successful controllers. The current implementation therefore exceeds the success rate of our earlier bipeds [10], where only a small fraction (about 10 percent) of evolutionary runs resulted in stable walking – and of these the required length was typically in excess of 100 generations. While it has to be taken into account that the two approaches used different physics engines (MathEngine SDK 1.0.5 vs. MathEngine Dynamics Toolkit 0.0.5), and a quantitative comparison is hence problematic, the improved implementation presented here can nevertheless clearly be considered superior in terms of evolutionary efficiency.

Another area of improvement is the perceived realism of the body dynamics, when compared with actual human walking. While bipeds produced in our earlier experiments [10] often displayed unnatural walking styles, the gaits found in the current implementation appear consistently more humanoid. To further confirm this notion, body dynamics data produced by our bipeds will have to be compared to human motion capture data.

Preliminary analysis suggests that in particular the free swinging legs constrain the dynamics of walking in a favourable manner. Walking styles that did not make use of a full lower leg swing (and hence the knee cap) were less stable than those which did employ it – similarly humans tend to fully extend their lower legs,

irrespective of the walking speed.

Experiments on simulated bipedal walking with combined active and passive actuation have recently been carried out elsewhere [20]. We believe, however, that the neuro-evolutionary method presented here confers crucial advantages. Firstly, it is fully automated, which brings about increased flexibility with respect to the body implementation. Different mass distributions, body dimensions, and actuator or joint types can be tested for their usefulness (e.g. in terms of their passive dynamics) simply by evolving suitable controllers (which is a matter of minutes on a modern PC). Unlike other approaches, no manual re-tuning of controller parameters is necessary.

Moreover, the CPG architecture used in this research allows the integration of sensory (in particular proprioceptive) feedback and tonic input into the controller. While this has yet to be implemented for the biped presented here, previous work by Taga [38] on planar bipedal walking has demonstrated that appropriate modulation of CPG activity increases the robustness of walking gaits on uneven terrain. We expect to achieve similarly positive results of a corresponding set-up in our 3D-physical simulation.

7 Conclusion

It was demonstrated that the evolution of straight line bipedal walking on even terrain is facilitated by the employment of appropriate passive dynamics, actuator types and controller architectures. In combination with incremental evolution, the length of evolutionary runs could be reduced to substantially less than 100 generations.

Walking gaits produced using this method are of an anthropomorphic nature – a result which is particularly due to the use of passive dynamics.

While the present paper addresses the evolution of straight line bipedal walking on even terrain, future implementations will incorporate sensory (in particular proprioceptive) information. The opinion is held that the neuro-evolutionary approach used here provides an ideal basis for these extensions.

8 Acknowledgements

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